

Tidal Inversion

implemented in Matlab (code available)

Shallow water equations in the frequency domain

$$\nabla \cdot gH\Omega^{-1}\nabla\zeta - i\omega\zeta = 0$$

open boundary forcing:

$$\zeta|_{OBC} = \zeta_{TPXO_{GLOBAL}}$$

$$\text{where } \Omega = \begin{bmatrix} i\omega + \kappa & -f \\ f & i\omega + \kappa \end{bmatrix}$$

Inverse solution found as:

$$\zeta(x, y) = \zeta_0(x, y) + \beta^T \mathbf{r}(x, y)$$

where

$$\mathbf{M}^* \boldsymbol{\alpha}_k = \boldsymbol{\delta}_k$$

Adjoint of
dynamics

$$\mathbf{M} \mathbf{r}_k = \mathbf{B} \boldsymbol{\alpha}_k$$

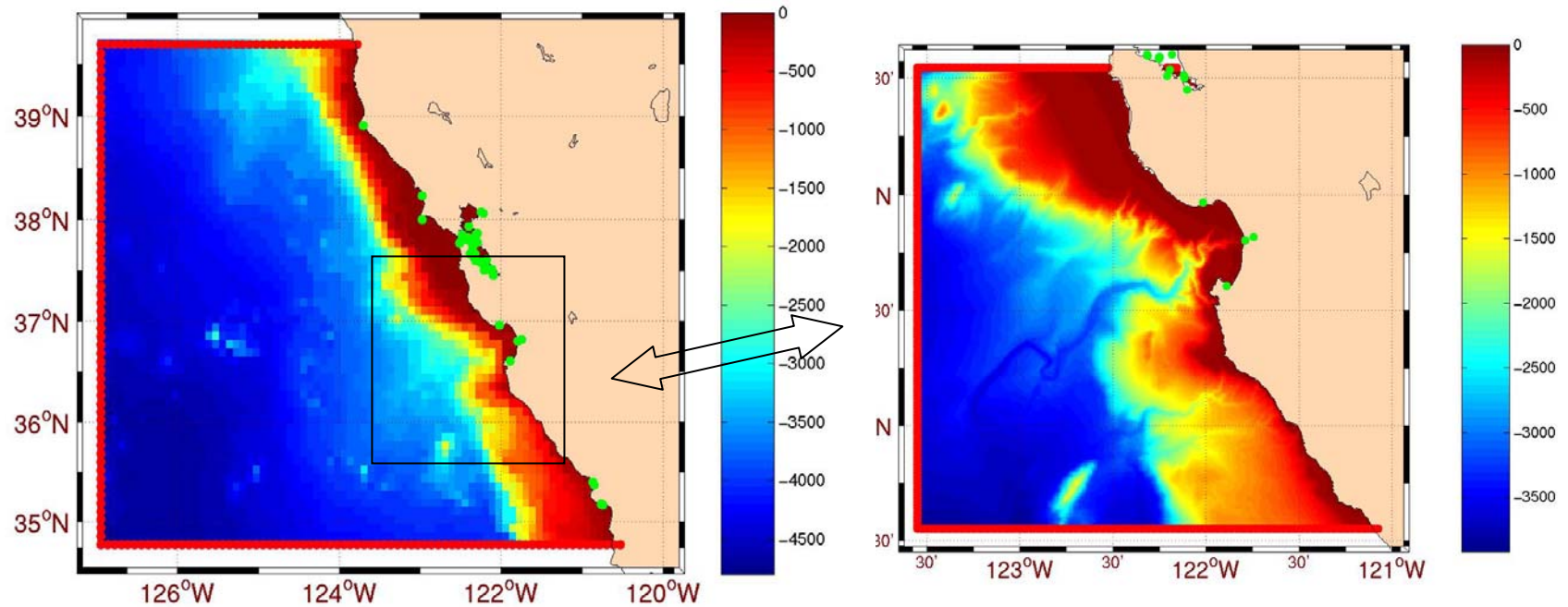
Dynamic error covariance

$$(\mathbf{C} + \mathbf{R})\boldsymbol{\beta} = \mathbf{y} - \mathbf{H}\zeta_0$$

Observational
error covariance

$$\mathbf{C}_{m \times m} = \left[\mathbf{Hr}_1 | \mathbf{Hr}_2 | \dots | \mathbf{Hr}_m \right]$$

Available Tide Gauges



Bottom Topography [m]

Monterey Bay Area Pacific Basin

Major Tidal Constituents

The main tidal constituents in the Monterey Bay Area:

M2 - Principal lunar semidiurnal constituent

K1 - Lunar diurnal constituent

O1 - Lunar diurnal constituent

S2 - Principal solar semidiurnal constituent

	Amplitude [m]	Velocities [cm/s]
M2	~ 0.50	~ 10-15
K1	~ 0.35	~ 15-20
O1	~ 0.21	~ 5-10
S2	~ 0.13	~ 2-5

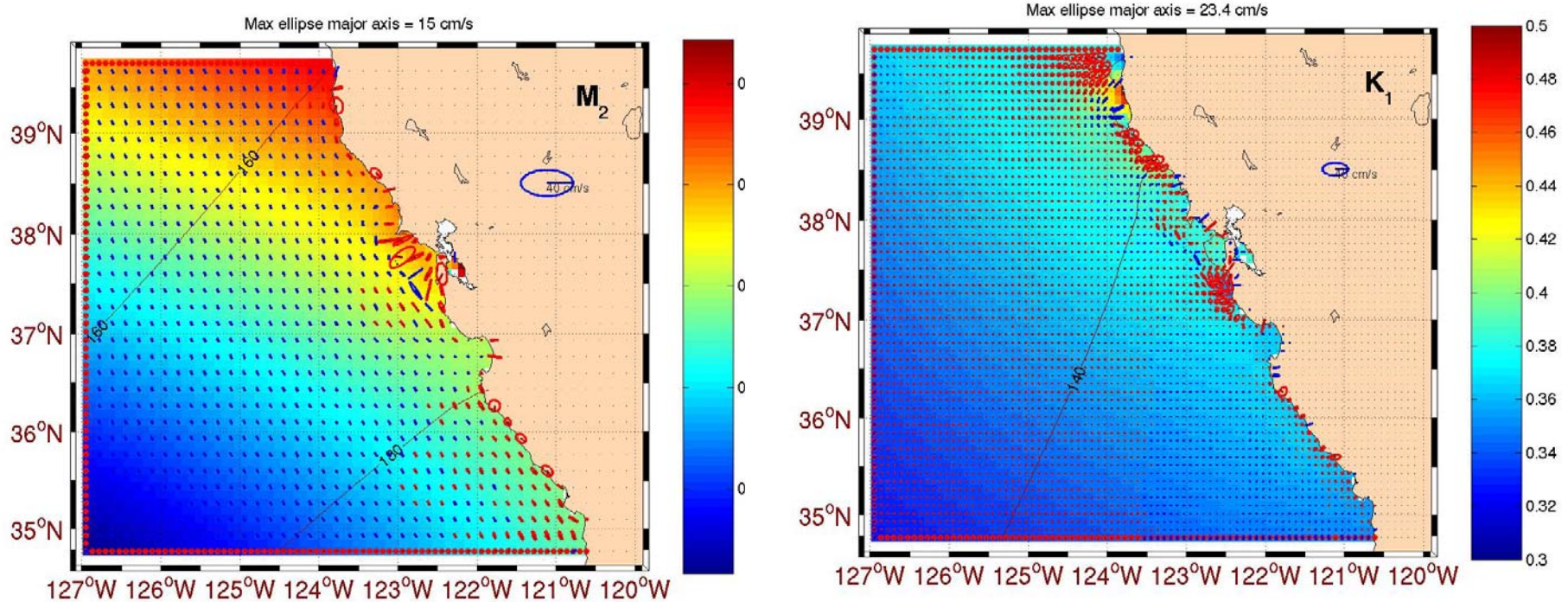
The computation was carried out for 8 constituents:

m2; s2; k1; o1; n2; p1; k2; q1;

Dynamics of diurnal and semi-diurnal constituents in the Monterey Bay Area

Diurnal => sub-inertial => form coastal trapped waves

Semi-diurnal => super-inertial => do not form coastal trapped waves

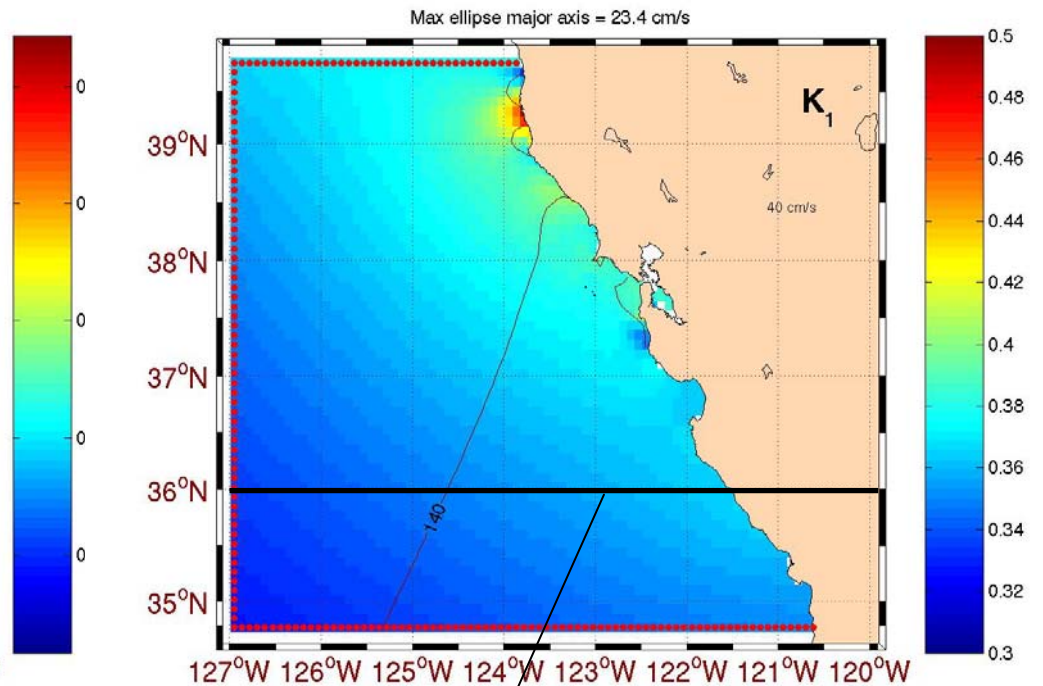
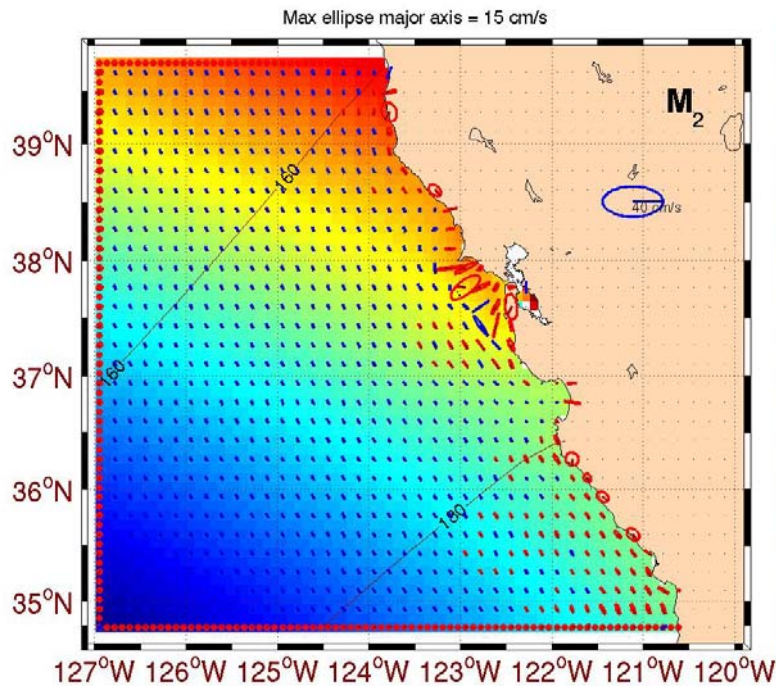


Monterey Bay Area Basin

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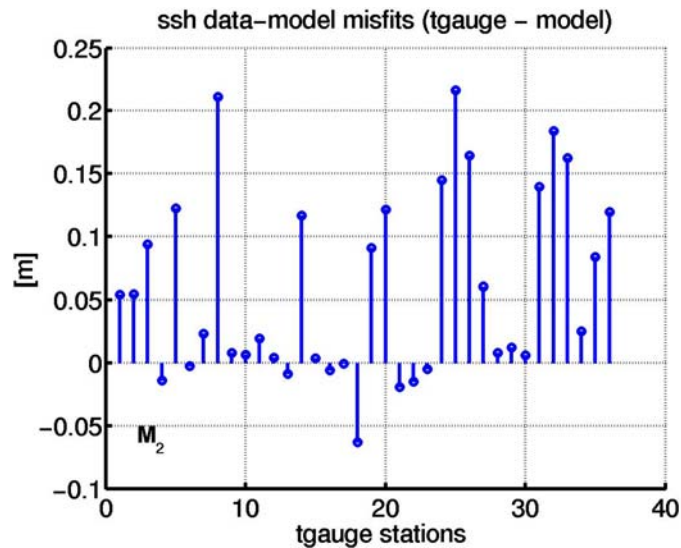


Same as before but vel. ellipses removed

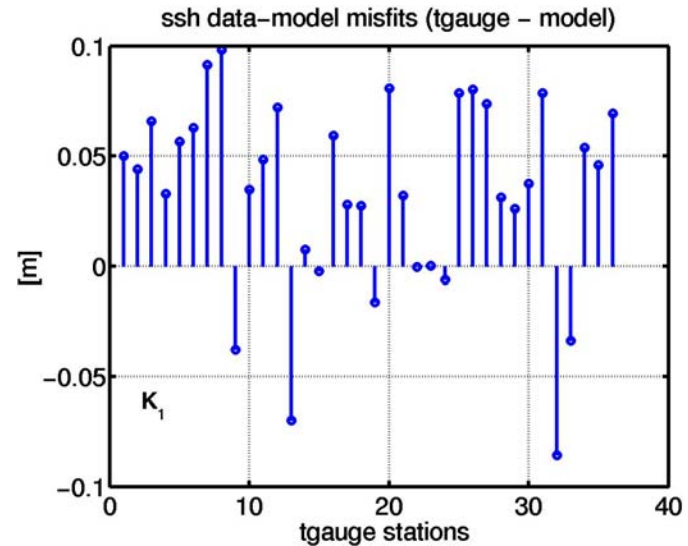
The latitude where K_1 becomes sub-inertial

Data-forward model misfits

M2

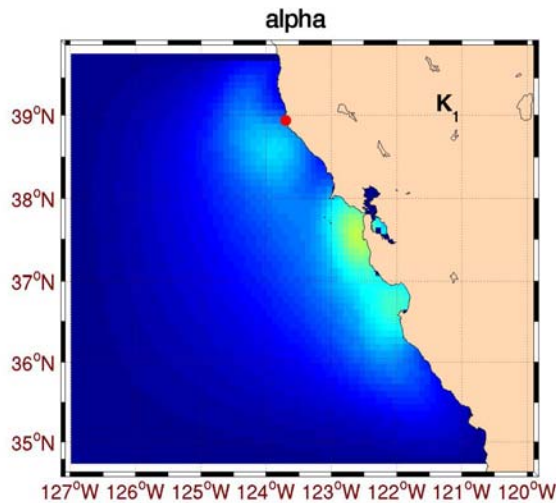


K1

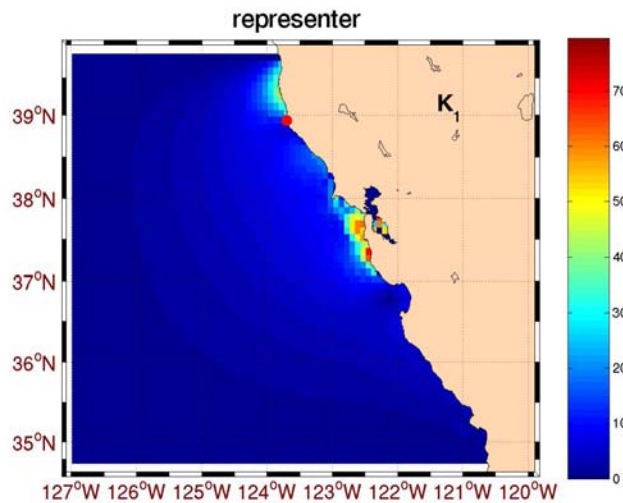


- model-data misfits:
 - Subscale processes
 - Baroclinic structure disregarded (important over sloping topography)
 - Model and observational errors

Adjoint variables and representers:



K1



Adjoint Variable

$$\mathbf{M}^* \boldsymbol{\alpha}_k = \boldsymbol{\delta}_k$$

Representer

$$\mathbf{M} \mathbf{r}_k = \mathbf{B} \boldsymbol{\alpha}_k$$

■ Difficulties related to the representer method:

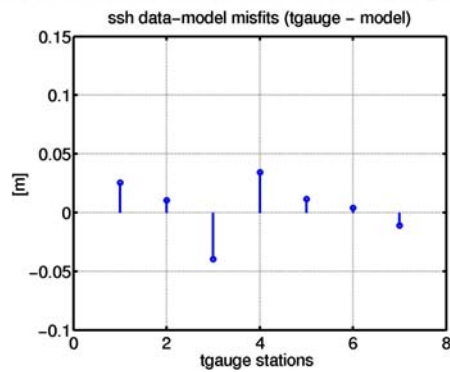
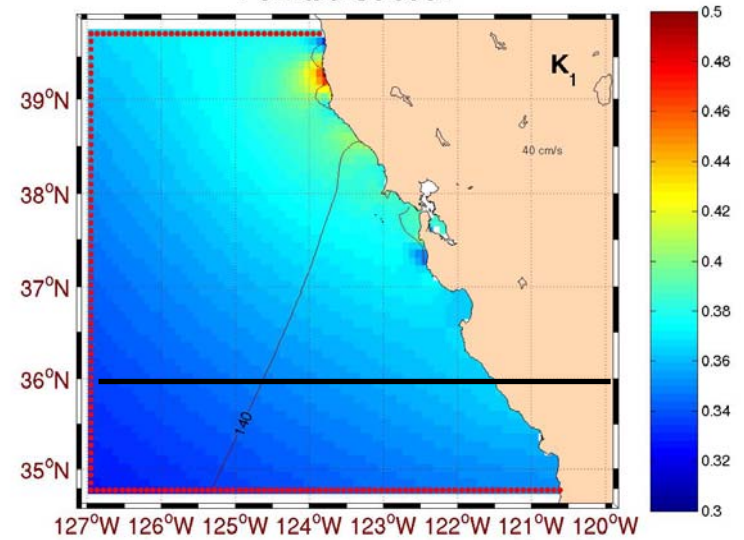
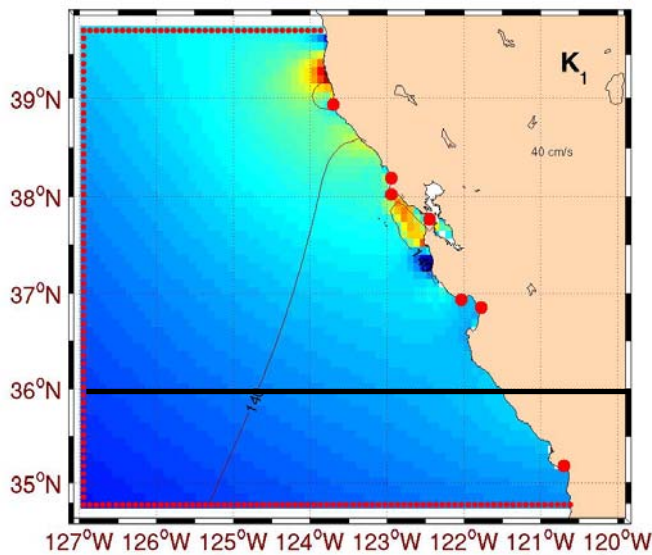
- Typically, representers are highly correlated with each other
- Maximum in a representer function might be far removed from the observation location
- Unknown dynamical error covariance

Inverse vs. Forward Solution

Inverse
Inverse Solution

K₁

Forward
Forward Solution



Data-
model
misfits

